ALG III 12/8/17 Final Exam Review
1.   Solve each equation.

a.
$$4(7-x)=39-3(x+5)$$
$$28-4x=39-3x-15$$
$$28-4x=24-3x$$
$$4=x$$

b.
$$11|x+6|=33$$
$$|x+6|=3$$
$$x+6=3 \qquad x+6=-3$$
$$x=-3 \qquad x=-9$$

c.
$$|x-21|+10=1$$
$$|x-21|=-9$$
abs can't be negative
No Solutions

d.
$$|x-4|=|x-6|$$
$$x-4=x-6 \qquad x-4=-(x-6)$$
$$-4=-6 \qquad x-4=-x+6$$
no solution $\qquad 2x=10$
$$x=5$$

2.   Solve each inequality. Represent your answer in INTERVAL NOTATION.

a.
$$4x-3<5x+2$$
$$-5<x$$
$$(-5,\infty)$$

b.
$$\frac{-4x+22}{5}\geq-3x$$
$$-4x+22\geq-15x$$
$$22\geq-11x$$
$$-2\leq x$$
$$[-2,\infty)$$

c.
$$3<4x-9\leq19$$
$$3<4x-9 \qquad\qquad 4x-9\leq19$$
$$12<4x \qquad AND \qquad 4x\leq28$$
$$3<x \qquad\qquad x\leq7$$
$$(3,7]$$

d.
$$k+7<-6 \qquad\qquad -2k\leq-18$$
$$k<-13 \qquad AND \qquad k\geq9$$
No Solution

e.
$$9x+3<-6 \qquad 3x-7>5$$
$$9x<-9 \quad or \quad 3x>12$$
$$x<-1 \qquad\qquad x>4$$
$$(-\infty,-1)\cup(4,\infty)$$

f.
$$|x+5|<8$$
$$x+5<8 \qquad x+5>-8$$
$$x<3 \quad and \quad x>-13$$
$$(-13,3)$$

g.

$$3\left|5x-8\right|+6>12$$
$$3\left|5x-8\right|>6$$
$$\left|5x-8\right|>2$$

$$5x-8>2 \qquad 5x-8<-2$$
$$5x>10 \quad or \quad 5x<6$$
$$x>2$$
$$x<\frac{6}{5}$$

$$\left(-\infty,\frac{6}{5}\right)\cup\left(2,\infty\right)$$

$$3\left|5x-8\right|+6>12$$ h.

$$\left|8x+6\right|+10\geq2$$
$$\left|8x+6\right|\geq-8$$

Absolute value will always be positive so

$$\left(-\infty,\infty\right)$$

3.   During 2nd quarter, John has scored 82%, 90%, 76% and 81% on his Algebra III tests. What must he earn on the fifth test for his average score to be at least 85%?

$$\frac{82+90+76+81+x}{5}\geq85$$
$$82+90+76+81+x\geq425$$
$$329+x\geq425$$
$$x\geq96$$
$$\left[96,\infty\right)$$

must score at least 96 percent.

4.   Emily mixed together 9 gal. of Brand A fruit drink and 8 gal. of Brand B fruit drink which contains 48% fruit juice. Find the percent of fruit juice in Brand A if the mixture contained 30% fruit juice.
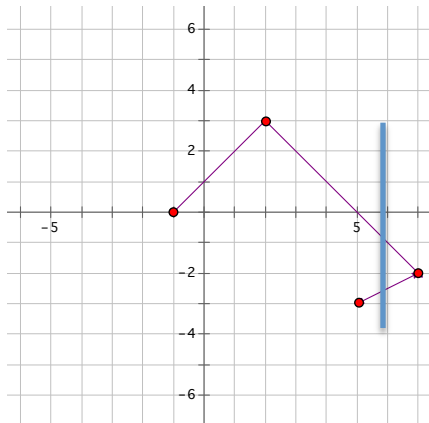
$$9x+8\left(0.48\right)=17\left(0.30\right)$$
$$9x+3.84=5.1$$
$$9x=1.26$$
$$x=0.14$$

14% solution

5.   Determine if the relation represents a function. State the domain and range.

a.

b.

X	Y
8	4
10	3
-4	2
-12	1
8	0

Function: YES/NO

Domain:_____[-1,7] _____

Range:_____[-3,3]_____
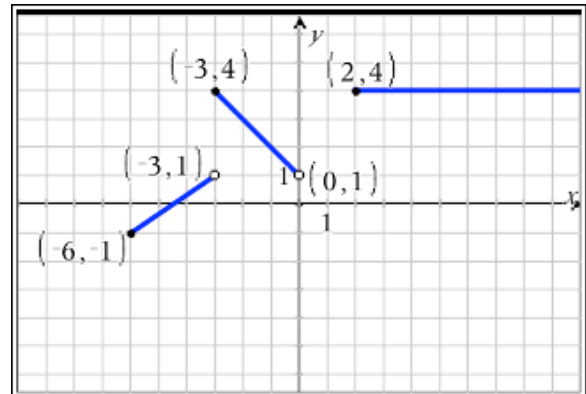
Function: YES/NO

Domain:____{-12,-4,8,10}_____

Range:____{0,1,2,3,4}_____

6.   Given the graph

a. Does the graph represent a function?
    How do you know?

b. Write an equation.

$$y = \begin{cases} \dfrac{2}{3}x+2 \text{ for } -6 \leq x < -3 \\ -x+1 \text{ for } -3 \leq x < 0 \\ 4 \text{ for } x \geq 2 \end{cases}$$

c. Find $f(2)$.   Plug in to third equation so y = 4

d. State the domain and range. Use the correct notation!

D: $\left[-6,0\right) \cup \left[2,\infty\right)$ R: $\left[-1,4\right]$

e. On what intervals is the graph increasing and decreasing?

Increasing: (-6,-3) Decreasing (-3,0)

7. Write a linear equation that is perpendicular to the line $5x - 2y = 10$ and passes through the point $(-3, 7)$.

Slope will be the opposite reciprocal of the slope of the line $5x - 2y = 10$

$5x - 2y = 10$
$-2y = 10 - 5x$
$y = -5 + \dfrac{5}{2}x$

so: $y - 7 = -\dfrac{2}{5}(x + 3)$
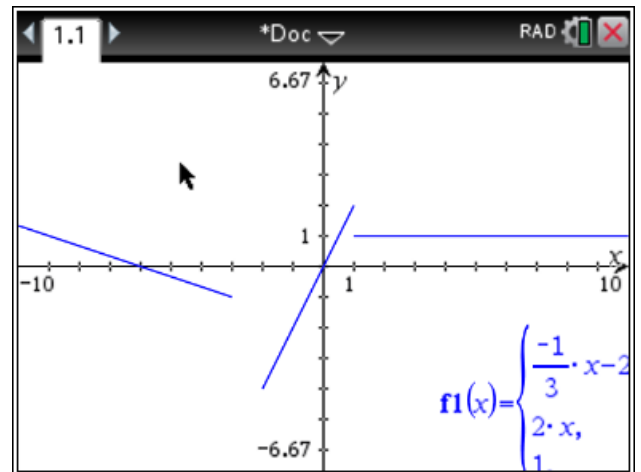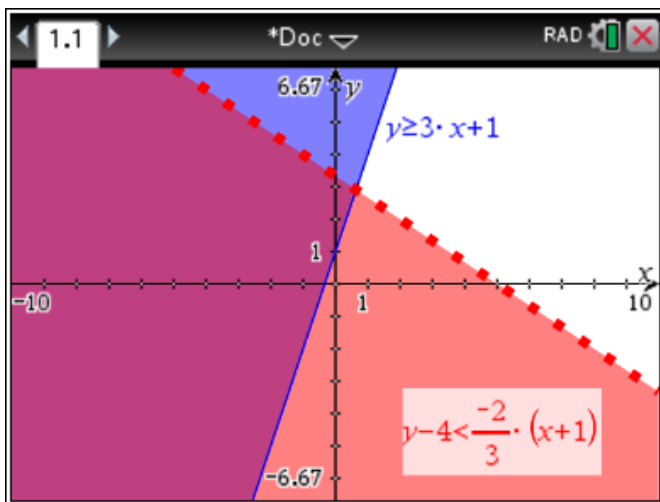
8. Write the equation of a horizontal line that passes through $(-1,\ 10)$.

y=10

9. Graph

a. $\begin{cases} y \geq 3x + 1 \\ y - 4 < -\dfrac{2}{3}(x+1) \end{cases}$

b. $f(x) = \begin{cases} -\dfrac{1}{3}x - 2 & for & x \leq -3 \\ 2x & for & -2 \leq x \leq 1 \\ 1 & for & x > 1 \end{cases}$

10. Given $f(x) = x^2 - 5$ and $g(x) = -2x + 8$

Find:

a.
$$f(x) = x^2 - 5$$
$$f(2) = 2^2 - 5$$
$$= 4 - 5$$
$$= -1$$

b.
$$g(x) = -2x + 8$$
$$g(0) = -2(0) + 8$$
$$= 8$$

c.
$$f(x) = x^2 - 5$$
$$f(a+2) = (a+2)^2 - 5$$
$$= a^2 + 4a + 4 - 5$$
$$= a^2 + 4a - 1$$

d.
$$(f+g)(x) = f(x) + g(x)$$
$$= x^2 - 5 + -2x + 8$$
$$= x^2 - 2x + 3$$

e.
$$(f \bullet g)(x) = f(x) \cdot g(x)$$
$$= (x^2 - 5)(-2x + 8)$$
$$= -2x^3 + 8x^2 + 10x - 40$$

f.
$$(g - f)(x) = g(x) - f(x)$$
$$= (-2x + 8) - (x^2 - 5)$$
$$= -2x + 8 - x^2 + 5$$
$$= -x^2 - 2x + 13$$

g.
$$(f + g)(-1) = (-1)^2 - 2(-1) + 3$$
$$= 1 + 2 + 3$$
$$= 6$$

h.
$$(f - g)(0) = f(0) - g(0)$$
$$= (0^2 - 5) - (02(0) + 8)$$
$$= -5 - 8$$
$$= -13$$

i.
$$\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$$
$$= \frac{(0^2 - 5)}{(02(0) + 8)}$$
$$= \frac{-5}{8}$$

j.
$$(g \bullet f)(4) = g(4)f(4)$$
$$= (-2(4)+8)(4^2-5)$$
$$= (0)(11)$$
$$= 0$$

k.
$$(g \circ f)(3) = g(f(3))$$
$$= g(3^2-5)$$
$$(g \circ f)(3) \qquad = g(4)$$
$$= -2(4)+8$$
$$= 0$$

l.
$$(g \circ g)(2) = g(g(2))$$
$$= g(-2(2)+8)$$
$$= g(4)$$
$$= -2(4)+8$$
$$= 0$$

m.
$$(f \circ g)(-1) = f(g(-1))$$
$$= f(-2(-1)+8)$$
$$= f(10)$$
$$= 10^2-5$$
$$= 95$$

n.
$$(g \circ f)(x) = g(f(x))$$
$$= g(x^2-5)$$
$$= -2(x^2-5)+8$$
$$= -2x^2+10+8$$
$$= -2x^2+18$$

o.
$$(f \circ f)(x) = f(f(x))$$
$$= f(x^2-5)$$
$$= (x^2-5)^2-5$$
$$= x^4-10x^2+25-5$$
$$= x^4-10x^2+20$$

11.   Simplify completely. Be sure to show all your work.

a. $$x^5 y^3 \cdot x^{-2} y^6 = x^{5-2} y^{3+6}$$
$$= x^3 y^9$$

b. $$\left(x^3\right)^2 = x^{3*2} = x^6$$

c. $$\frac{x^{10}}{x^3} = x^{10-3} = x^7$$

d. $$\left(2xy^4\right)^3 = 2^3 x^3 y^{4*3} = 8x^3 y^{12}$$

e. $$\left(\frac{5}{x^7}\right)^{-1} = \frac{x^7}{5}$$

f. $$\frac{1}{5^{-2}} = 5^2 = 25$$

g. $$\frac{3^2}{3^{-2}} = 3^{2--2} = 3^4 = 81$$

h. $$\frac{2x^{-3} y^4 \cdot 6x^6 y}{4x^4 y^2} = \frac{12}{4} x^{-3+6-4} y^{4+1-2} = 3x^{-1} y^3 = \frac{3y^3}{x}$$

i. $$\left(\frac{4p^3 q^{-5}}{pq^{-1}}\right)^{-2} = \frac{4^{-2} p^{-2*3} q^{-2*-5}}{p^{-2*1} q^{-2*-1}}$$
$$= \frac{16^{-1} p^{-6} q^{10}}{p^{-2} q^2} = \frac{1}{16} p^{-6--2} q^{10-2}$$
$$= \frac{1}{16} p^{-8} q^8 = \frac{q^8}{16p^8}$$

j. $$\frac{27x^8 y^2}{3xy^{11}} = \frac{27}{3} x^{8-1} y^{2-11}$$
$$= 9x^7 y^{-9} = \frac{9x^7}{y^9}$$

12.   Divide.

a. $$\left(25x^5 - 15x^4 + 5x^3 - 30x^2 + 55x - 100\right) \div (5x)$$

$$5x \overline{\smash{\big)}\, 25x^5 - 15x^4 + 5x^3 - 30x^2 + 55x - 100}$$

quotient: $5x^4 - 3x^3 + 1x^2 - 6x + 11 - \dfrac{20}{x}$

b. $\left(2x^3 + 9x^2 + x - 12\right) \div \left(2x + 3\right)$

$$
\begin{array}{r}
x^2 + 3x - 4 \\
2x+3\overline{)2x^3 + 9x^2 + x - 12} \ ] \\
2x^3 + 3x^2 \\
6x^2 + x \\
6x^2 + 9x \\
\text{-8x-12} \\
\text{-8x-12} \\
0
\end{array}
$$

c. $\left(8x^4 + 16x^3 - 26x^2 - 8x + 3\right) \div \left(x + 3\right)$

$$
\begin{array}{r}
8x^3 - 8x^2 - 2x - 2 + \dfrac{9}{x+3} \\
x+3\overline{)8x^4 + 16x^3 - 26x^2 - 8x + 3} \\
8x^4 + 24x^3 \\
\text{-}8x^3 - 26x^2 \\
\text{-}8x^3 - 24x^2 \\
\text{-}2x^2 - 8x \\
\text{-}2x^2 - 6x \\
-2x + 3 \\
-2x - 6 \\
9
\end{array}
$$

13.   Factor.

a. $9x^2 - 25$
$\left(3x\right)^2 - 5^2$
$\left(3x - 5\right)\left(3x + 5\right)$

b. $x^2 + 14x - 15$
$\left(x + 15\right)\left(x - 1\right)$

c. $x^2 - x - 56$
$\left(x - 8\right)\left(x + 7\right)$

d. $2x^2 - 8x + 3x - 12$
$2x\left(x - 4\right) + 3\left(x - 4\right)$
$\left(2x + 3\right)\left(x - 4\right)$

e.
$$4x^2 - 12x + 9$$
$$(2x-3)^2$$

f.
$$x^3 + 2x^2 - 9x - 18$$
$$x^2(x+2) - 9(x+2)$$
$$(x^2-9)(x+2)$$
$$(x-3)(x+3)(x+2)$$

14.   Solve.

a.
$$2x(2x-18)(x+7)=0$$

$$2x=0 \quad \begin{matrix} 2x-18=0 \\ 2x=18 \end{matrix} \quad \begin{matrix} x+7=0 \\ x=-7 \end{matrix}$$
$$x=0 \qquad x=9$$

b.
$$x^2 + 7x = 30$$
$$x^2 + 7x - 30 = 0$$
$$(x-3)(x+10)=0$$

$$x-3=0 \quad x+10=0$$
$$x=3 \qquad x=-10$$

c.
$$2x+x+3=0$$
$$(2x+3)(x+1)=0$$

$$\begin{matrix} 2x+3=0 \\ 2x=-3 \\ x=-\dfrac{3}{2} \end{matrix} \quad \begin{matrix} x+1=0 \\ x=-1 \end{matrix}$$

d.
$$2x^2 - 30x + 108 = 0$$
$$2(x^2 - 15x + 54) = 0$$
$$2(x-9)(x-6)=0$$

$$x-9=0 \quad x-6=0$$
$$x=9 \qquad x=6$$